AD-

7798

UNCLASSIFIED

Best Available Copy

Aeroballistic Research Report 104

ON THE ATTENUATION OF THE SHOCK WAVE ABOUT AN AXIALLY-SYMMETRIC BODY

ABSTRACT: It is shown theoretically that the pressure rise across the shock wave about a body of revolution decays asymptotically as the inverse three-fourths power of the radial distance from the body axis. Experimental data is presented to support the theory.

U. S. NAVAL ORDNANCE LABORATORY
White Oak, Maryland

The purpose of this investigation was to obtain information on the decay of shock waves about missiles.

This investigation was performed under task NOL-Re3d-453-1-52.

EDWARD L. WOODYARD
Captain, USN
Commander

H. H. KURZWEG, Chief
Aeroballistic Research Department
By direction

CONTENTS

ILLUSTRATIONS

ON THE ATTENUATION OF THE SHOCK WAVE ABOUT AN AXIALLY-SYMMETRIC BODY

INTRODUCTION

1. The attenaution of the shock wave about a two-dimensional body is discussed by Munk and Crown in reference (a). It is shown that the difference between the shock angle and the free-stream Mach angle decays inversely as the square root of the distance away from the body axis. Hence, the shock wave contour at large distances from the body becomes parabolic, having no asymptote. In addition to practical considerations such as the extrapolation of a given portion of a shock wave to obtain wave drag, as indicated in reference (a), the question of whether or not a shock wave about an axially-symmetric body has an asymptote and the nature of the shock decay are of fundamental importance.

2. In 1949 the author was interested in the problem of the antentuation of shock wave about a body of revolution and in discussion with various people was verbally informed that some work had been done on this problem which showed that:

$$x = r\sqrt{M_1^{\,2}-1} - ar^{1/4} \tag{1}$$

where x = streamwise coordinate
r = radial distance from body axis
M_1 = free-stream Mach number
a = an arbitrary constant

3. The work containing the formula as well as the derivation could not be located in the literature. A preliminary check of this formula with the then available experimental data did not yield satisfactory agreement. In order to evaluate the correctness of the formula the author of this report made an independent analysis which is presented herein. The results yielded a formula essentially the same as given above. Substantiation of the formula was obtained when recent accurate and reliable data were used.

4. This report contains the author's derivation of the theoretical formula for the attenuation of the shock wave about a body of revolution (or any three-dimensional body, for that matter) and a presentation of experimental data to substantiate it.

ANALYSIS

The Characteristic Equations:

5. If the shock wave curvature is small, as it is as large distances from a body, then with little error we may take the flow behind the shock

to be isentropic. The characteristic equations then take the form

$$d\nu \mp d\theta \mp \left(\frac{\sin\theta \sin\alpha}{\sin(\alpha \pm \theta)}\right)\frac{dr}{r} = 0 \tag{2}$$

where

ν is the expansion angle
θ is the flow inclination
α is the Mach angle
r is the radial distance from the body axis, and

the upper and lower signs refer respectively to families I and II characteristics as indicated in Figure 1. When the difference from free-stream conditions is small, the characteristic equations may be approximated by

$$d\nu \mp d\theta \mp \frac{\theta}{r}\,dr = 0 \tag{3}$$

6. From the geometry of Figure 1, it can be seen that

$$\frac{dr_{II}}{dr_I} = \frac{\sin(\alpha-\theta)\,\sin(\alpha+\theta-\sigma)}{\sin(\alpha+\theta)\,\sin(\alpha+\sigma-\theta)} \tag{4}$$

or

$$\frac{dr_{II}}{dr_I} \simeq \frac{(\alpha-\alpha_1)+\theta-\varepsilon}{\sin 2\alpha_1} \tag{5}$$

where $\varepsilon = \sigma - \alpha_1$

σ is the shock wave angle

and the subscript "1" refers to free-stream conditions.

For small differences

$$\alpha - \alpha_1 = \left(\frac{d\alpha}{d\nu}\right)_1 (\nu - \nu_1) \tag{6}$$

where

$$\left(\frac{d\alpha}{d\nu}\right)_1 = -\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{M_1^2-1}\right) \tag{7}$$

and from the condition of isentropy immediately behind the shock wave we have the relation

$$\nu_1 - \nu = \theta \tag{8}$$

From the weak oblique shock formulas we find

$$\theta \simeq \frac{4}{\gamma+1}\left(\frac{M_1^2-1}{M_1^2}\right)\varepsilon \tag{9}$$

Substituting these formulas into equation (5), we obtain

$$\frac{dr_{II}}{dr_I} \simeq \frac{\varepsilon}{\sin 2\alpha_1} \tag{10}$$

Thus it can be seen that dr_{II} is one order of magnitude smaller than dr_I. Ignoring dr_{II} in the characteristic equations for the II family, we find

$$d\nu + d\theta = 0 \tag{11}$$

or

$$\nu_1 - \nu = \theta \tag{8}$$

at any point behind the shock where conditions are not far from free-stream values. Substituting equation (8) into the characteristic equation for family I, we obtain

$$2\,d\theta + \frac{\theta}{r}\,dr = 0 \tag{12}$$

or upon integration

$$\frac{\theta}{\theta'} = \sqrt{\frac{r'}{r}} \tag{13}$$

where the primed values represent the constants of integration. This equation is the one upon which the asymptotic nature of the shock wave is based.

The Shock Wave:

7. The slope of the Mach lines of family I is given by

$$\frac{dr}{dx} = \tan\phi \tag{14}$$

where

$$\phi = \alpha + \theta \tag{15}$$

For small deviations from free-stream, we can write, as before,

$$\phi - \alpha_1 = \left(\frac{\gamma+1}{2} \cdot \frac{M_1^2}{M_1^2 - 1} \right) \theta = \theta' \sqrt{\frac{r'}{r}} \tag{16}$$

where we now take x', r' to be the origin of the expansion waves which weaken the shock. This point may be real or imaginary. For a cone-cylinder model, this point is at the shoulder. However, the actual existence of such a point is immaterial to asymptotic behavior of the shock wave.

8. Using the first two terms of a Taylor's Expansion, we may write the slope, (14), as

$$\frac{dr}{dx} = \tan \alpha_1 \left[1 + \frac{M_1^2}{\sqrt{M_1^2 - 1}} (\phi' - \alpha_1) \sqrt{\frac{r'}{r}} \right] \tag{17}$$

Let

$$K = \frac{M_1^2}{\sqrt{M_1^2 - 1}} (\phi' - \alpha_1) \sqrt{r'} \tag{18}$$

which is constant along a given Mach line. Then equation (17) becomes

$$dr - \tan \alpha_1 \, dx = \frac{K}{K + \sqrt{r}} \, dr \tag{19}$$

and upon integration, yields the equation of the shock wave:

$$(r_s - r') - (x_s - x') \tan \alpha_1 = \int_{r'}^{r_s} \frac{K}{K + \sqrt{r}} \, dr \tag{20}$$

Where the subscript "s" refers to conditions immediately behind the shock wave. Now the slope of the shock wave is given by

$$\frac{dr_s}{dx_s} = \tan \sigma = \tan (\alpha_1 + \varepsilon) \tag{21}$$

or
$$\frac{dx_s}{dr_s} = \cot\alpha_1 - M_1^2 \varepsilon \tag{22}$$

Differentiation of equation (20) yields another expression for the slope of the shock wave:

$$\cot\alpha_1 - \frac{dx_s}{dr_s} = \cot\alpha_1 \left[\left(\frac{K}{K+\sqrt{r_s}} \right) + \left(\frac{dK}{dr_s} \right) \int_{r'}^{r_s} \frac{d}{dK} \left(\frac{K}{K+\sqrt{r}} \right) dr \right] \tag{23}$$

Combining with (22) we have

$$\frac{M_1^2}{\sqrt{M_1^2-1}} \varepsilon = \frac{\frac{K}{\sqrt{r_s}}}{1+\frac{K}{\sqrt{r_s}}} + \frac{dK}{dr_s} \int_{r'}^{r_s} \frac{d}{dK} \left(\frac{K}{K+\sqrt{r}} \right) dr \tag{24}$$

From (18)

$$K = \frac{M_1^2}{\sqrt{M_1^2-1}} (\phi' - \alpha_1) \sqrt{r'} \tag{18}$$

or

$$K = \frac{M_1^2}{\sqrt{M_1^2-1}} (\phi_s - \alpha_1) \sqrt{r_s} \tag{25}$$

From the foregoing equations, it can be shown that

$$\phi_s - \alpha_1 \simeq 2\varepsilon \tag{26}$$

and therefore

$$K \simeq \frac{2 M_1^2}{\sqrt{M_1^2-1}} \varepsilon \sqrt{r_s} \tag{27}$$

Thus the quantity

$$\frac{\frac{K}{\sqrt{r_s}}}{1+\frac{K}{\sqrt{r_s}}} \simeq \frac{2 M_1^2}{\sqrt{M_1^2-1}} \varepsilon \tag{28}$$

Differentiating equation (27) we obtain

$$\frac{dK}{dr_s} = \frac{2M_1^2}{\sqrt{M_1^2-1}}\sqrt{r_s}\left(\frac{\mathcal{E}}{2\sqrt{r_s}} + \frac{d\mathcal{E}}{dr_s}\right) \tag{29}$$

The integral in equation (24) may be evaluated as follows:

$$\int_{r'}^{r_s} \frac{d}{dK}\left(\frac{K}{K+\sqrt{r}}\right)dr = \int_{r'}^{r_s} \frac{\sqrt{r}}{(K+\sqrt{r})^2}\,dr$$

$$= 2\left[(K+\sqrt{r}) - 2K\ln(K+\sqrt{r}) - \frac{K^2}{K+\sqrt{r}}\right]_{r'}^{r_s} \simeq 2\sqrt{r_s}\left(1 - \frac{2M_1^2}{\sqrt{M_1^2-1}}\mathcal{E}\ln\frac{r_s}{r'}\right) \tag{30}$$

Substituting these relations into equation (24) and dropping the subscript "s", we obtain for the equation of the shock wave the relation

$$\frac{d\mathcal{E}}{dr} = -\frac{3}{4}\frac{\mathcal{E}}{r}\left[\frac{1 - \frac{4}{3}\frac{M_1^2}{\sqrt{M_1^2-1}}\mathcal{E}\ln\frac{r}{r'}}{1 - 2\frac{M_1^2}{\sqrt{M_1^2-1}}\mathcal{E}\ln\frac{r}{r'}}\right] \tag{31}$$

This differential equation can be satisfied in the region $r \longrightarrow \infty$ only with $\mathcal{E}\ln r \longrightarrow 0$

The quantity $\frac{M_1^2}{\sqrt{M_1^2-1}}$ remains finite for $1 < M_1 < \infty$.

Considering $\frac{M_1^2}{\sqrt{M_1^2-1}}\mathcal{E}\ln\frac{r}{r'}$ as a higher order infinitesimal and integrating equation (31), we obtain a formula for the asymptotic shape of the shock wave:

$$\mathcal{E} = \frac{k}{r^{3/4}} \tag{32}$$

where k is the constant of integration which may be taken equal to

$$\left(\mathcal{E}_0 / r_0^{3/4}\right)$$

and x_0, r_0 is a point at which $\mathcal{E}$ is known.

9. Equation (32) is the one we have been seeking. It can be seen to be equivalent to equation (1) by differentiating the latter.

10. From equation (1) or (32), it follows that while the shock angle approaches the free-stream Mach angle as $r \to \infty$, the shock wave contour itself does not have an asymptote.

11. Furthermore, from equations 4.27 and 4.29 of reference (b), it follows that the fractional pressure rise across a weak shock wave is given by

$$\frac{\Delta p}{p_1} = \frac{4\gamma}{\gamma+1}\sqrt{M_1^2-1}\;\varepsilon \tag{33}$$

This pressure disturbance decays also as the inverse three-fourths power of the distance from the body axis.

12. In order to demonstrate the validity of the three-fourths power law, equation (32), it would seem logical to plot experimental values of ε vs r on log-log graph paper. The theoretical relation, which is the asymptotic solution for $r \to \infty$, then appears as a straight line with a negative slope of 3/4. This comparison is given in the following section.

COMPARISON WITH EXPERIMENT

13. The data presented herein were obtained from spark shadowgraphs of firings made in the NOL Pressurized Ballistics Range. The velocity was determined accurately electronically and the sound speed obtained from temperature measurements; thus the Mach number could be calculated accurately. It is believed that the greatest source of error was incurred in the measurement of the shock angles which was done on Bausch and Lomb comparators. The scatter of the data is thought to be indicative of the measuring accuracy and this appears to be within satisfactory limits.

14. Figure 2 shows a comparison of theory and experiment for the shock wave about a sphere. The agreement can be seen to be very good. Figure 3 shows a similar comparison at a higher Mach number. It should be emphasized that equation (32) is an asymptotic solution and hence the divergence of the theoretical curve from the experimental data at small distances from the body is not contradictory. At large distances from the body the asymptotic solution appears to be approached very closely.

15. Figure 4 illustrates the attenuation of the shock wave for a missile. The theoretical curve again agrees very well with the experimental data. Some of the irregularities at large radii may be due to the influence of the fins.

16. The data for another missile is presented in Figure 5. Here, too, the agreement with theory is close up to about 15 body radii away from the

missile axis at which point the shock strength appears to remain constant. This effect is believed due to yaw of the missile. That this is the explanation might have been demonstrated by measuring the shock angles for the other side of the missile. However, in order to obtain shock waves which extend to as large a distance from the body as possible, plates were selected on which the missile was off-center. Hence no such comparison could be made. It seems obvious, however, that this constant shock angle cannot be maintained indefinitely if the drag is to remain finite. Thus, when the expansion waves from the tail of the missile begin to reach the shock, it will begin to decay once more as the inverse three-fourths power of the distance.

CONCLUSION

17. It is believed that the experimental data presented herein confirm the basic validity of equation (32) for the asymptotic attenuation of the shock wave about a body of revolution.

FURTHER HISTORICAL NOTE

18. After the completion of this report and just prior to publication several pertinent references were brought to the attention of the author. The unspecified reference alluded to in the introduction was found to be the work of Whitham, reference (c). Therein, a derivation of the formula in question is given. However, prior to him, DuMond et al in reference (d) gave a different derivation and presented experimental data for large distances from the missile. Furthermore, as far as the author has been able to determine, the formula was first mentioned by Landau in reference (e) where only a simple statement of the formula is given.

19. The present report with new experimental data was considered to be of sufficient interest to justify publication in addition to the preceeding reports.

REFERENCES

(a) Munk, M. M. and Crown, J. C. The Head Shock Wave. Proceedings VII International Congress for Applied Mechanics, Vol II_2, London 1948

(b) Liepmann, H. W. and Puckett, Allen E. Introduction to Aerodynamics of a Compressible Fluid Wiley, 1947

(c) Whitham, G. B. The Behavior of Supersonic Flow Past a Body of Revolution, Far from the Axis Proceedings of the Royal Society of London, Vol. 201, No. 1064, March 1950, p. 89

(d) DuMond, Jesse W. M., Cohen, E. R., Panofsky, W. K. H., and Deeds, E. A Determination of the Wave Forms and Law of Propagation and Dissipation of Ballistic Shock Waves Journal of the Acoustic Society Of America, Vol. 18, No. 1, July 1946

(e) Landau, L. D. On Shock Waves Journal of Physics of the Academy of Sciences of the USSR, Vol. 6, pp 229-230, 1942

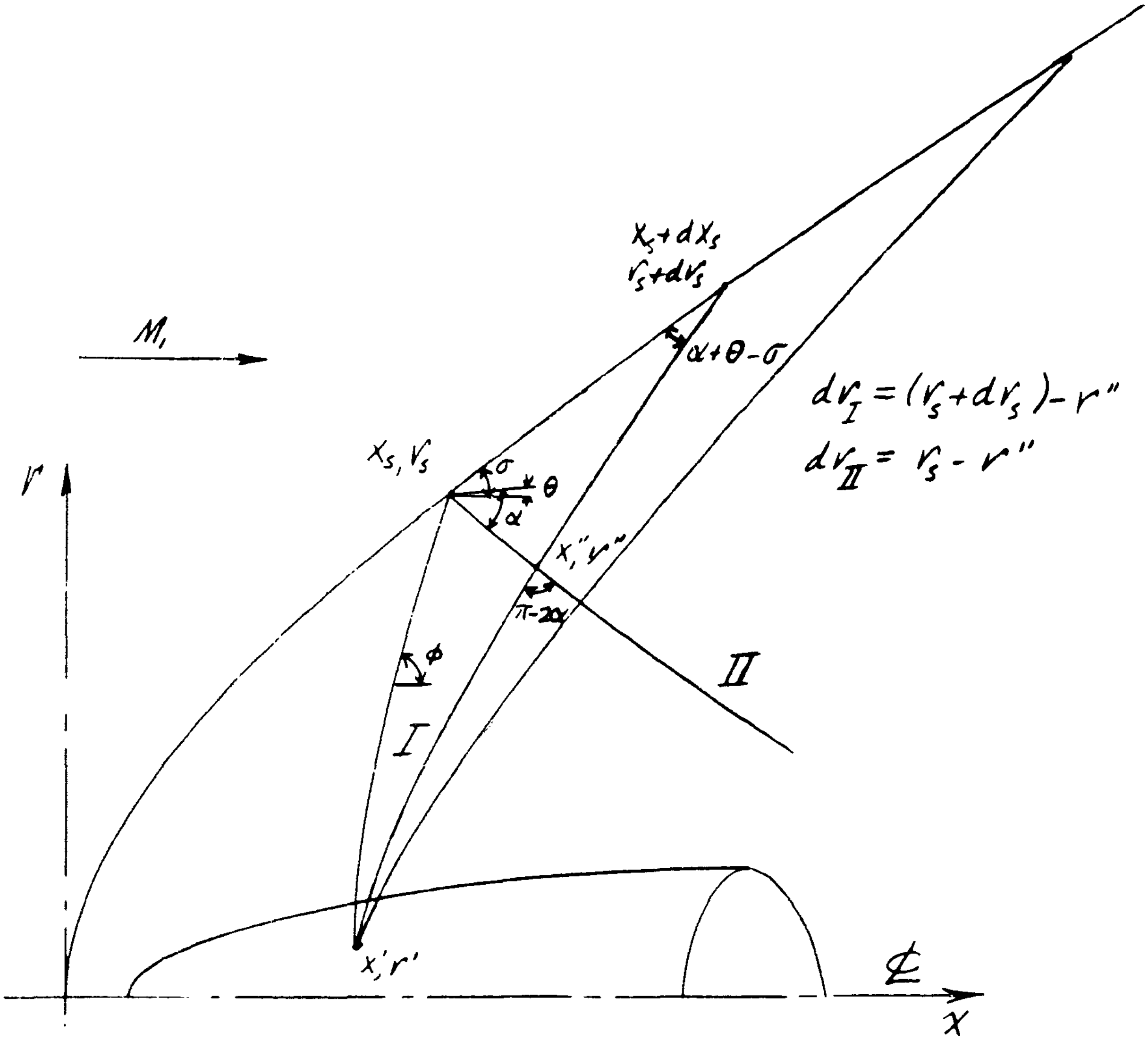

FIGURE 1:- SKETCH OF CHARACTERISTIC SYSTEM

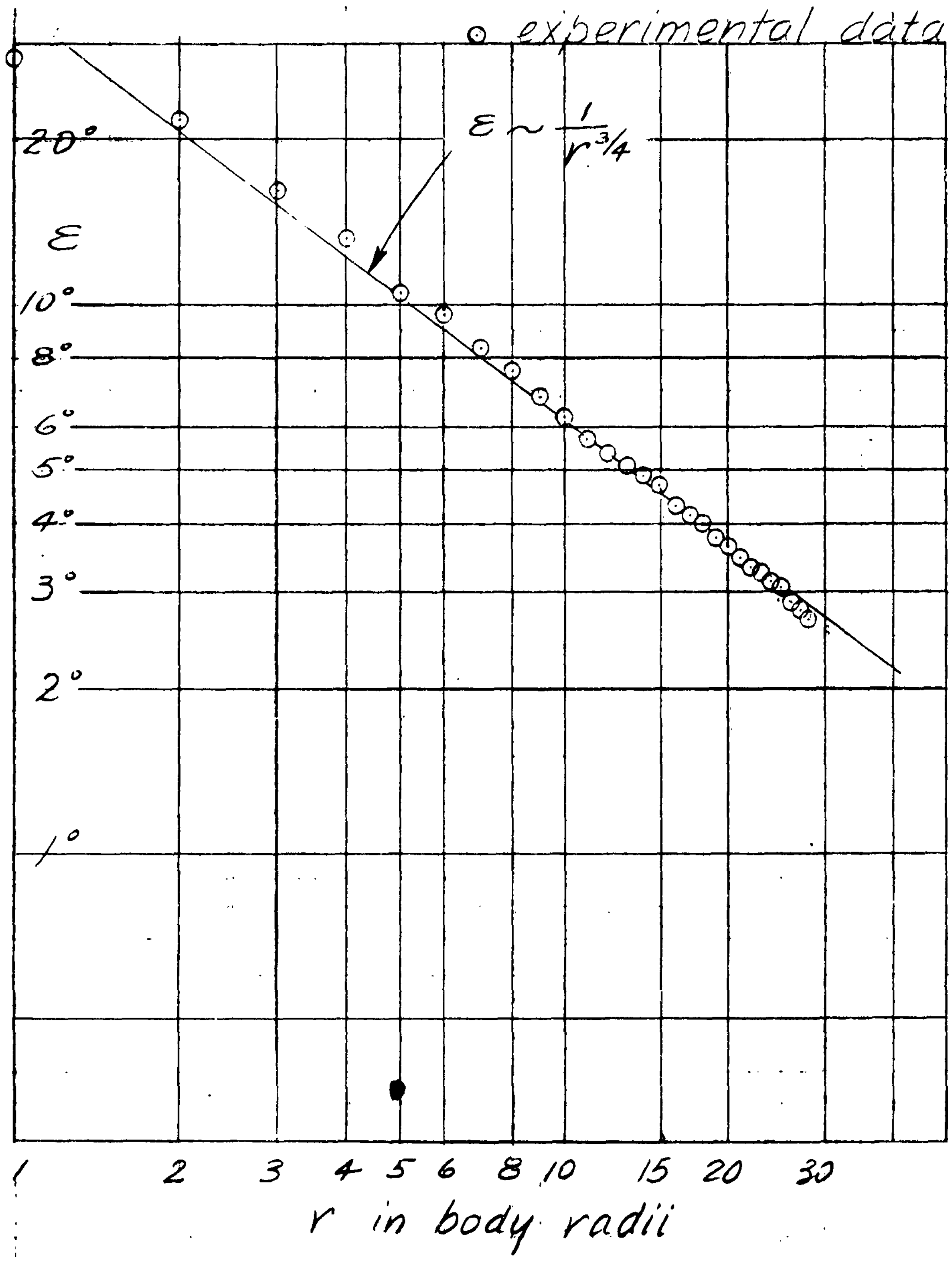

FIGURE 2 :- SHOCK WAVE ATTENUATION FOR A SPHERE AT $M = 1.260$

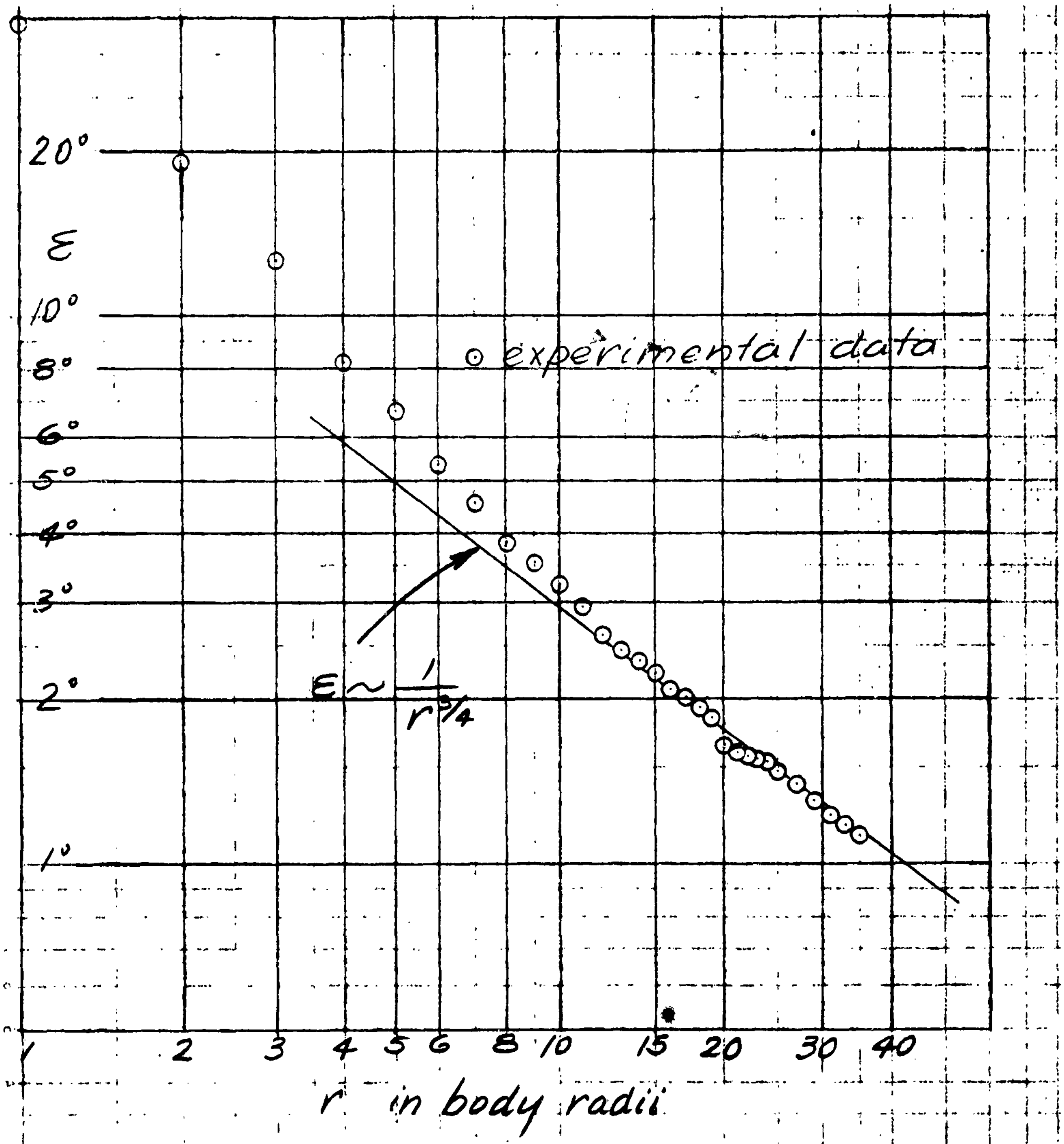

FIGURE 3:- SHOCK WAVE ATTENUATION FOR A SPHERE AT M = 2.673

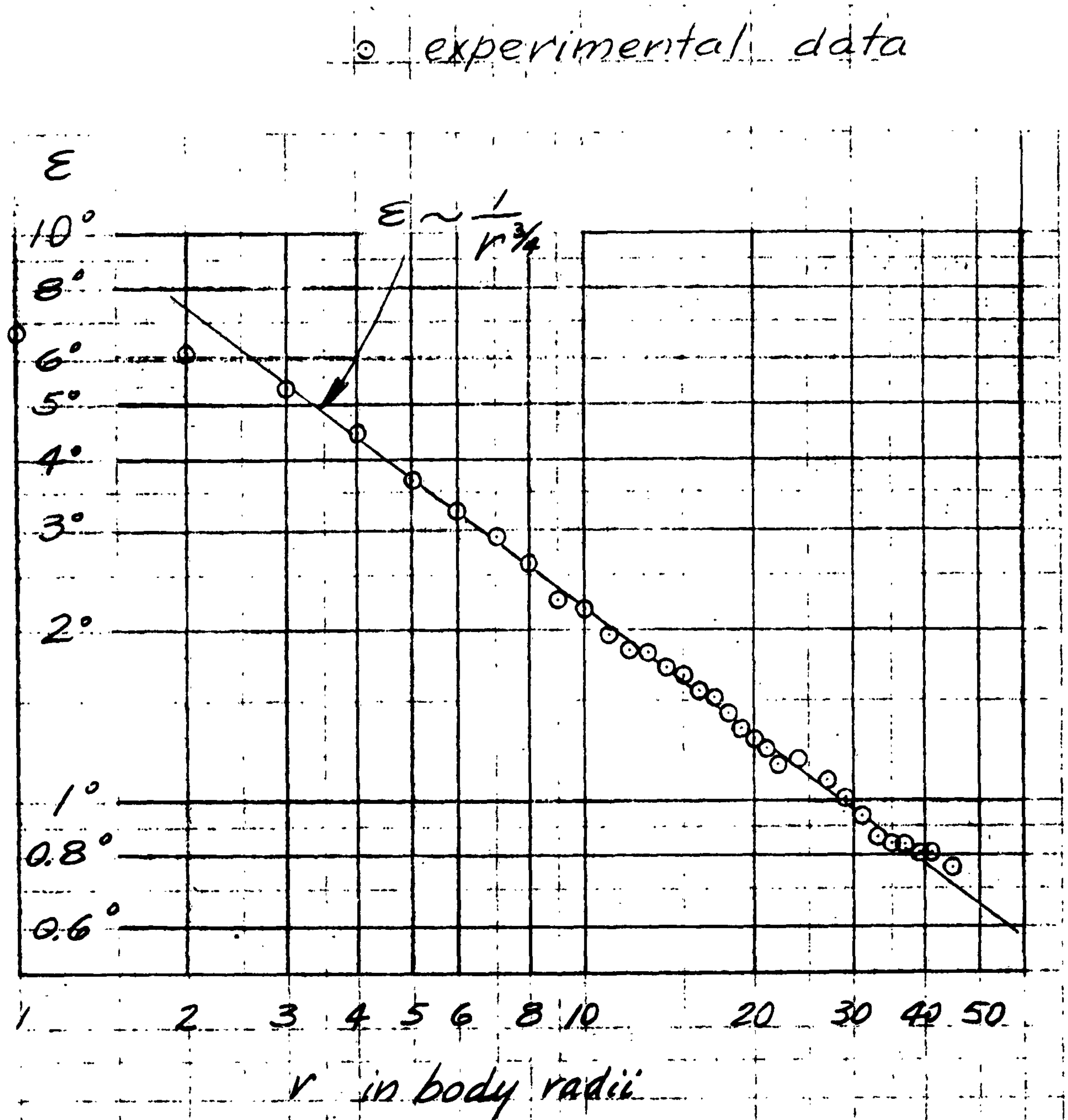

FIGURE 4:- SHOCK WAVE ATTENUATION FOR A MISSILE AT M=1.855

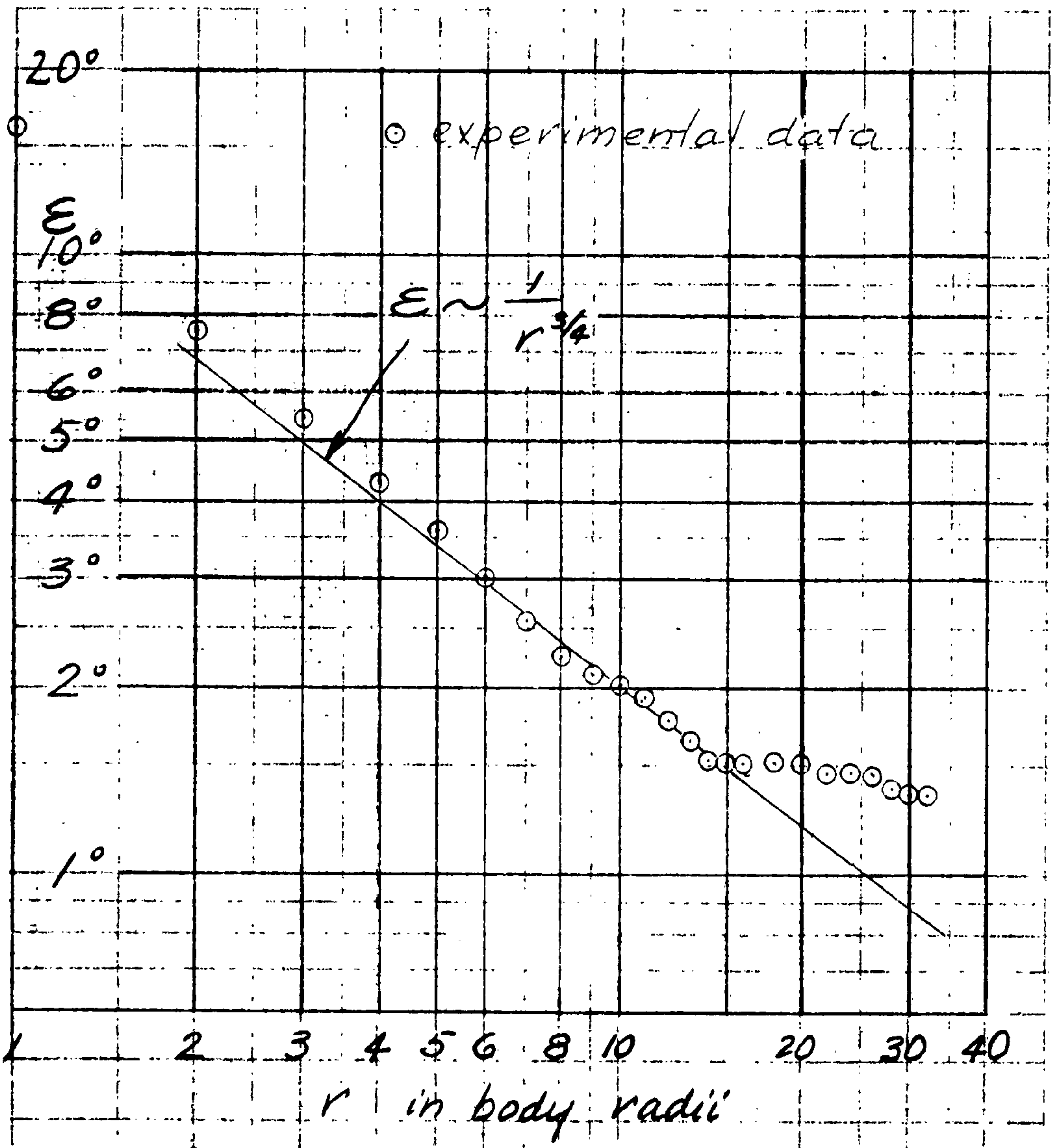

FIGURE 5 :- SHOCK WAVE ATTENUATION FOR A MISSILE AT M = 1.578

Aeroballistic Research Department
External Distribution List for Development (X2)

No. of Copies	
	Chief, Bureau of Ordnance Department of the Navy Washington 25, D. C.
1	Attn: Rea
1	Attn: Rexe
1	Attn: Re3d
3	Attn: Re9a
	Chief, Bureau of Aeronautics Department of the Navy Washington 25, D. C.
1	Attn: AER-TD-414
2	Attn: RS-7
	Commander U.S. Naval Ordnance Test Station Inyokern P.O. China Lake, California
2	Attn: Technical Library
	Commander U.S. Naval Air Missile Test Center Point Mugu, California
3	Attn: Technical Library
	Superintendent U.S. Naval Postgraduate School Monterey, California
1	Attn: Librarian
	Commanding Officer and Director David Taylor Model Basin Washington 7, D. C.
1	Attn: Hydrodynamics Laboratory
	Chief of Naval Research Navy Research Section Library of Congress
2	Washington 25, D. C.
	Chief, Naval Operations Department of the Navy Washington 25, D. C.
1	Attn: Op-51
	Office of Naval Research Department of the Navy Washington 25, D. C.
2	Attn: Code 463
	Director Naval Research Laboratory Washington 25, D. C.
1	Attn: Code 2021 Code 3800
	Officer-in-Charge Naval Aircraft Torpedo Unit U.S. Naval Air Station
1	Quonset Point, Rhode Island
	Office, Chief of Ordnance Washington 25, D. C.
1	Attn: ORDTU

No. of Copies	
	Research and Development Board Library Branch Pentagon 3D1041
2	Washington 25, D. C.
	Chief, AFSWP P.O. Box 2610 Washington, D. C.
1	Attn: Technical Library
	Chief, Physical Vulnerability Branch Air Targets Division Directorate of Intelligence Headquarters, USAF
1	Washington 25, D. C.
	The Artillery School Antiaircraft and Guided Missiles Br. Fort Bliss, Texas
2	Attn: Research and Analysis Sec.
	Commanding General Wright Air Development Center Wright-Patterson Air Force Base Dayton, Ohio
8	Attn: WCAPD
2	Attn: WCRR
1	Attn: WCSD
5	Attn: WCSO
	Director Air University Library
1	Maxwell Air Force Base, Alabama
	Commanding General Aberdeen Proving Ground Aberdeen, Maryland
1	Attn: C.L. Poor
	National Bureau of Standards Washington 25, D. C.
1	Attn: Librarian (Ord. Dev. Div.)
1	Attn: W. Ramberg
	National Bureau of Standards Corona Laboratories (Ord.Dev.Div.) Corona, California
1	Attn: Dr. H. Thomas
	National Bureau of Standards Building 3U, UCLA Campus 405 Hilgard Avenue Los Angeles 24, California
1	Attn: Librarian
	University of California Berkeley 4, California
1	Attn: Mr. G. J. Maslach
1	Attn: Dr. S. A. Schaaf
	California Inst. of Technology Pasadena 4, California
2	Attn: Librarian(Guggenheim Aero. Lab.) VIA: BuAero

No. of Copies	
	University of Michigan Willow Run Research Center Ypsilanti, Michigan
1	Attn: L.R. Biasell VIA: InsMat
	University of Minnesota Rosemount, Minnesota
1	Attn: J. Leonard Frame VIA: Ass't InsMat
	The Ohio State University Columbus, Ohio
1	Attn: G.L. VonEschen VIA: Ass't InsMat
	Polytechnic Institute of Brooklyn 99 Livingston Street Brooklyn 2, New York
1	Attn: Dr. Antonio Ferri VIA: ONR Branch Office
	Princeton University Forrestal Research Center Library Project Squid
1	Princeton, New Jersey
	Massachusetts Inst. of Technology Project Meteor Cambridge 39, Massachusetts
2	Attn: Guided Missiles Library
	Applied Physics Laboratory The Johns Hopkins University 8621 Georgia Avenue Silver Spring, Maryland
1	Attn: Arthur G. Norris Technical Reports Office VIA: NIO
	Armour Research Foundation 35 West 33rd Street Chicago 16, Illinois
1	Attn: Engr. Mechanics Division VIA: ONR Branch Office
	Defense Research Laboratory University of Texas Box 1, University Station
1	Austin, Texas VIA: InsMat
	Eastman Kodak Company Navy Ordnance Division 50 West Main Street Rochester 4, New York
2	Attn: Dr. Herbert Trotter, Jr. VIA: NIO

No. of Copies	
	General Electric Company Building #1, Campbell Avenue Plant Schenectady, New York
1	Attn: J.C. Hoffman VIA: InsMachinery
	The Rand Corporation 1500 Fourth Street Santa Monica, California
1	Attn: The Librarian VIA: InsMat
	Consolidated Vultee Corporation Daingerfield, Texas
1	Attn: J.E. Arnold, Manager VIA: Dev. Contract Office
	Douglas Aircraft Company, Inc. 3000 Ocean Park Boulevard Santa Monica, California
1	Attn: Mr. E.F. Burton VIA: BuAero Resident Rep.
	North American Aviation, Inc. 12214 Lakewood Boulevard Downey, California
2	Attn: Aerophysics Laboratory VIA: BuAero Rep.
	National Advisory Committee for Aero. 1724 F Street Northwest Washington 25, D. C.
5	Attn: Mr. E. B. Jackson
	Ames Aeronautical Laboratory Moffett Field, California
1	Attn: H. Julian Allen
2	Attn: A.C. Charters
	Theoretical Aerodynamics Division Langley Aeronautical Laboratory Langley Field, Virginia
1	Attn: Theoretical Aerodynamics Div.
1	Attn: Dr. A. Busemann
2	Attn: J. Steele
	NACA Lewis Flight Propulsion Lab. Cleveland Hopkins Airport Cleveland, Ohio
1	Attn: Dr. John C. Evvard
	Hughes Aircraft Company Culver City, California
1	Attn: Dr. Allen E. Puckett
	Institute of Aerophysics University of Toronto Toronto 5, Ontario
1	Attn: Dr. Gordon N. Patterson, Dir. VIA: BuOrd (Ad8)

Aeroballistic Research Department
External Distribution List for Development (X2a)

No. of Copies	
	National Bureau of Standards Washington 25, D. C.
1	Attn: G. B. Schubauer
	Graduate School of Aero Engr. Cornell University Ithaca, New York
1	Attn: W. R. Sears VIA: ONR Branch Office
	Applied Math. and Statistics Lab. Stanford University Stanford, California
1	Attn: Mr. R. J. Langle VIA: Ass't InsMat
	Harvard University 109 Pierce Hall Cambridge 38, Massachusetts
1	Attn: Prof. R. von Mises